Trigonometric Functions Using the Unit Circle Video Lecture

Section 6.2

Course Learning Objectives:
Demonstrate an understanding of trigonometric functions and their applications.

Weekly Learning Objectives:

1) **Find the coordinates of a terminal point.**
2) **Find a reference number for an angle.**
3) **Find the exact values of the trigonometric functions using a point on the unit circle.**
4) **Find the exact values of the trigonometric functions of quadrantal angles.**
5) **Find the exact values of the trigonometric functions of $\pi/4 = 45^\circ$.**
6) **Find the exact values of the trigonometric functions of $\pi/6 = 30^\circ$.**
7) **Find the exact values of the trigonometric functions of $\pi/3 = 60^\circ$.**
8) **Find the exact values of the trigonometric functions for integer multiples of $\pi/4 = 45^\circ$, $\pi/6 = 30^\circ$, and $\pi/3 = 60^\circ$.**
9) **Use a calculator to approximate the value of a trigonometric function.**

Trigonometric Functions Using the Unit Circle

Consider a circle, radius 1, center at the origin.

Equation of circle:

Consider a mapping where we mark off a distance, t, around the unit circle starting at the point $(1, 0)$ and move counterclockwise if $t > 0$ and clockwise if $t < 0$. The point $P(x, y)$ that we arrive at is called the terminal point.

What is the terminal point of each of the following real numbers?

a) $t = \frac{\pi}{2}$

b) $t = -\frac{3\pi}{2}$

c) $t = \pi$

d) $t = -6\pi$

Find the coordinates of the terminal point if $t = \frac{\pi}{4}$.

Use symmetry of the unit circle to find the terminal point for each of the following.

a) $t = -\frac{5\pi}{4}$

b) $t = \frac{7\pi}{4}$

c) $t = \frac{3\pi}{4}$

Find the coordinates of the terminal point if $t = \frac{\pi}{6}$.

Use symmetry of the unit circle to find the terminal point for each of the following.

a) $t = -\frac{7\pi}{6}$

b) $t = \frac{11\pi}{6}$

c) $t = \frac{7\pi}{6}$

Use symmetry across the line $y = x$ and the terminal point for $t = \frac{\pi}{6}$ to find the coordinates of the terminal point for $t = \frac{\pi}{3}$.

Use symmetry of the unit circle to find the terminal point for each of the following.

a) $t = -\frac{7\pi}{3}$

b) $t = \frac{11\pi}{3}$

c) $t = \frac{2\pi}{3}$

As you can see from the above examples the terminal point of any number relates back to the "corresponding" terminal point in the first quadrant.

Definition: Let t be a real number. The ***reference number*** $\bar{t}$ *associated with t is the shortest distance along the unit circle between the terminal point determined by t and the x-axis.*
$\bar{t}$ measures the angle between t and the closest part of the $x-$ axis. It should be between 0 and $\frac{\pi}{2}$.

Find the reference numbers for each of the following:

a) $t = \dfrac{5\pi}{6}$

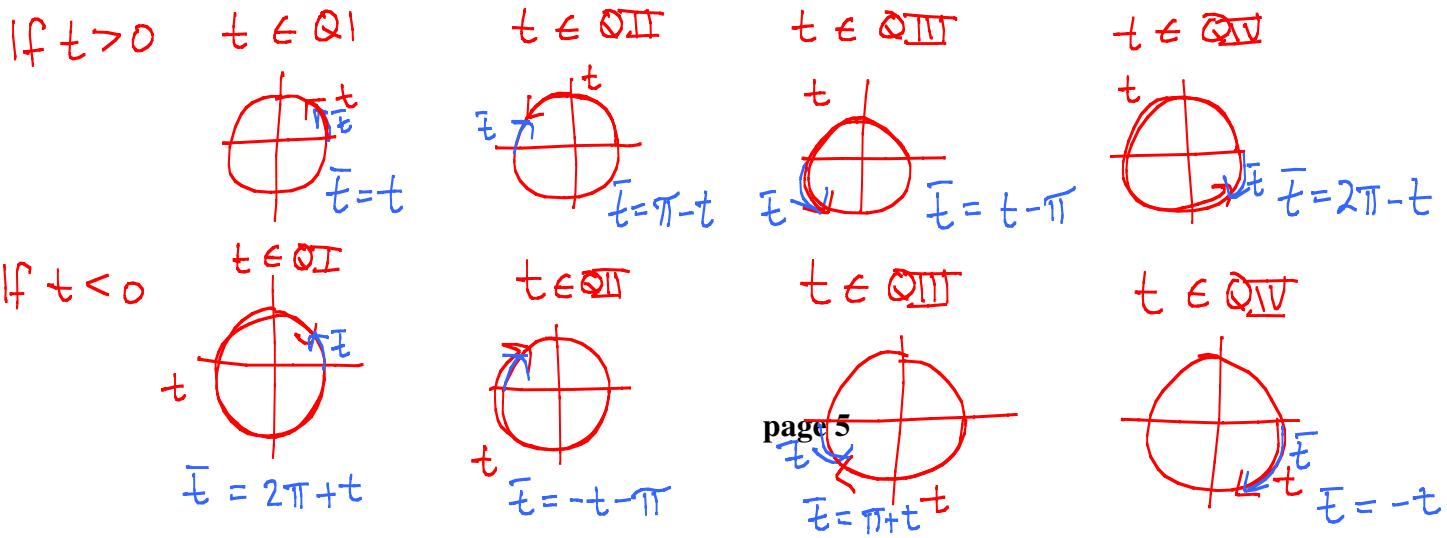
b) $t = \dfrac{5\pi}{4}$

c) $t = -\dfrac{5\pi}{3}$

d) $t = -2$

To find the terminal point for any number t,

(1) Find the reference number $\bar{t}$.
(2) Find the terminal point $Q(a, b)$ determined by $\bar{t}$.
(3) The terminal point determined by t is $P(\pm a, \pm b)$ where the signs are chosen according to the quadrant in which the terminal point lies.



It is essential that you memorize the coordinates of the special values of t given in the following table:

t	Terminal point determined by t
0	$(1, 0)$
$\frac{\pi}{6}$	$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\frac{\pi}{4}$	$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\frac{\pi}{3}$	$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\frac{\pi}{2}$	$(0, 1)$

Find the coordinates for each of the following using reference numbers.

a) $t = \frac{7\pi}{3}$

b) $t = \frac{13\pi}{6}$

c) $t = \frac{31\pi}{3}$

d) $t = -\frac{73\pi}{3}$

We can now define the six trigonometric functions:

Definition of the 6 trigonometric functions:

Let t be any real number and let $P(x, y)$ be the terminal point on the unit circle determined by t.

$\sin t = y$ $\quad$ $\cos t = x$ $\quad$ $\tan t = \dfrac{y}{x}\ (x \neq 0)$

$\csc t = \dfrac{1}{y}\ (y \neq 0)$ $\quad$ $\sec t = \dfrac{1}{x}\ (x \neq 0)$ $\quad$ $\cot t = \dfrac{x}{y}\ (y \neq 0)$

Find the value of each of the trigonometric functions at

a) $t = \dfrac{5\pi}{6}$

b) $t = \dfrac{\pi}{3}$

c) $t = -\dfrac{\pi}{2}$

The following table shows some special values of the 6 trigonometric functions. These values need to be memorized.

t	sin t	cos t	tan t	csc t	sec t	cot t
$\mathbf{0}$	0	1	0	—	1	—
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{2}$	1	0	—	1	—	0

If $P(x, y)$ is the terminal point for any number t, then θ is the angle in standard position measured in radians whose terminal side is the ray from the origin through P.

Since $r = 1$ on a unit circle, and $s = r\theta$ with the arc length $s = t$, then $t = 1 \cdot \theta$ or $t = \theta$.

Therefore,

sin θ = sin t cos θ = cos t tan θ = tan t

csc θ = csc t sec θ = sec t cot θ = cot t

Since the values of the trig functions of an angle θ are determined by the coordinates of the terminal point $P(x, y)$, the units on θ are irrelevant. θ can be in degrees or radians.

Find the exact value of each expression:

cos$(\frac{5\pi}{4})$ sin $135°$ tan $315°$

csc$(-60°)$ cot$(\frac{5\pi}{3})$ sec $210°$

cos$(48°)$ csc$(21°)$ tan$(\frac{5\pi}{12})$

cos (-1.62) cot (14.2) sec(8.14)