10.7 Dividing Polynomials

Objective A: Divide a polynomial by a monomial.

Dividing a Polynomial by a Monomial

Divide each term of the polynomial by the monomial.

\[
\frac{a + b}{c} = \frac{c \neq 0}
\]

Divide.

1. \[6x^3 + 3x^2 \text{ by } 3x\]

2. \[\frac{24x^6 - 12x^4 + 6x}{4x^2}\]

3. \[\frac{20x^5y + 16x^3y^3 - xy^2}{-2xy}\]
Objective B: Use long division to divide a polynomial by another polynomial.

1. Divide

\[ 23 \overline{) 4026} \]

2. Multiply

3. Subtract

4. Bring down

Dividend and divisor must be arranged in descending order.

1. Divide

\[ x - 2 \overline{) x^2 - 5x + 6} \]

2. Multiply

3. Subtract
   Draw the line and change the signs.

4. Bring down

\[ \frac{3x - 4 + 3x^2 + 4x^3}{x + 2} \]
\[
\frac{-5b + 6b^2 + 4}{3b - 1}
\]

\[
\frac{x^3 + 64}{x + 4}
\]
Synthetic Division

Use Synthetic Division to divide a polynomial by a binomial.

To use this short-cut the divisor must be in the form \(x - c\). In this example \(c = 3\).

\[ \begin{array}{cccc}
2 & 5 & 2 \\
1 - 3 & 2 - 6 & 5 - 13 & 5 - 15 \\
2x^3 - 6x^2 & 5x^2 - 13x & 5x^2 - 15x & \\
2x + 1 & 2x - 6 & \\
7 & \\
\end{array} \]

Coefficients of the dividend

\[ \begin{array}{cccc}
3 & 2 & -1 & -13 & +1 \\
6 & 2 & 5 & \\
\end{array} \]

Bring down the first coefficient of the dividend. Multiply by \(c\) and place the result under the second coefficient of the dividend. Add. Repeat the process.

\[ \begin{array}{cccc}
3 & 2 & -1 & -13 & +1 \\
6 & 15 & 6 & \\
2 & 5 & 2 & 7 & \\
\end{array} \]

Coefficients of the quotient remainder

The degree of the quotient is one less than the degree of the dividend.

Answer: \(\frac{2x^2 + 5x + 2 + \frac{7}{x-3}}{x-3}\)

What is the equation represented by the following synthetic division? \((a \div b = c)\) 5.7
Use synthetic division to divide.

1. \((x^2 + 2x + 4) ÷ (x + 3)\)

\[
\begin{array}{c|cccc}
4 & 2 & 0 & -35 & 12 \\
\hline
& 8 & 32 & -12 & \\
\hline
& 2 & 8 & -3 & 0 \\
\end{array}
\]

2. \((3x^3 - 2x^2 + 5x + 4) ÷ (x - 2)\)

3. \((x^3 + 1) ÷ (x + 1)\)

11.1 The Greatest Common Factor and Factoring by Grouping

Factoring – the process of writing a polynomial as a product. This is the reverse of multiplying.

The first step in factoring a polynomial is to see whether the terms of the polynomial have a common factor. If there is one, we can write the polynomial as a product by factoring out the common factor. We will usually factor out the greatest common factor (GCF).

Objective A: Find the greatest common factor of a list of integers.

The GCF of a list of integers is the largest integer that is a factor of all the integers.

**Finding the GCF of a List of Integers**

1. Write each number as a product of prime numbers.
2. Identify the common prime factors.
3. The product of all common prime factors found in Step 2 is the GCF. If there are no common factors, the GCF is 1.

**Ex.** 24 = 2 x 2 x 2 x 3

36 = 2 x 2 x 3 x 3

Form a product using the prime factors common to both numbers: 2 x 2 x 3 = 12; GCF = 12

**Ex.** 18 = 2 x 3 x 3

30 = 2 x 3 x 5

42 = 2 x 3 x 7

Form a product using the prime factors common to all three numbers: 2 x 3 = 6; GCF = 6

**Find the GCF:**

1. 15, 45
2. 32, 33
3. 30, 75
4. 12, 28, 40
Objective B: Find the greatest common factor of a list of terms.

The GCF of a list of common variables raised to powers is the variable raised to the smallest exponent in the list.

Ex. \( x^2 = x \cdot x \) \( x^4 = x \cdot x \cdot x \cdot x \) \( x^6 = x \cdot x \cdot x \cdot x \cdot x \cdot x \)

The common factors are \( x \cdot x \) which is the same as \( x^2 \) or the “smallest exponent of \( x \)”.

Find the GCF:

1. \( y, y^3, y^4 \)
2. \( x^4, x^5, x^6 \)

The GCF of a list of terms is the product of the GCF of the numerical coefficients and the GCF of the variable factors.

3. \( 25x^2, 10x^7 \)
4. \( 52y^4, 39y^3, 91y^4 \)
5. \( 35r^2s^5, -15r^3s^5, 21rs^3 \)

Objective C: Factor out the greatest common factor from the terms of a polynomial.

Factoring a Monomial from a Polynomial

1. Determine the greatest common factor of all terms in the polynomial.
2. Express each term as the product of the GCF and its other factor. \textit{You can obtain the other factor for each term by dividing each term by the GCF.}
3. Use the distributive property to factor out the GCF.

1. \( 4x + 10 \)
2. \( y^3 + 2y \)
3. \( 5x^3 + 10x^4 \)

4. \( -21x^3y - 49x^2y^2 \)
5. \( 8x^5y^4 - 16x^3y^3 - 24x^2y^2 \)
Objective D: Factor a polynomial by grouping

To Factor a Four-Term Polynomial by Grouping

1. Group the terms in two groups of two terms so that each group has a common factor.
2. Factor out the GCF from each group.
3. If there is now a common binomial factor in the groups, factor it out.
4. If not, rearrange the terms and try these steps again.

1. \(ab + b + 3a + 3\)  
2. \(12x^3 + 21x^2 - 20x - 35\)

3. \(4x^2 - 12xy - 7x + 21y\)  
4. \(6x^2 - 12xy - 10x + 20y\)

6. \(y(x + 3) + 2(x + 3)\)  
7. \(a(b^2 + 2) - (b^2 + 2)\)